Electricity and Magnetism, Exam 2, 12/03/2020

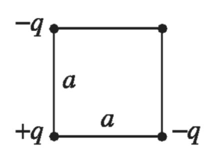
5 questions, 48 points

Write your name and student number on each answer sheet. Use of a calculator is allowed. You may make use of the provided formula sheet. The same notation is used as in the book, i.e. a bold-face **A** is a vector, $\hat{\boldsymbol{x}}$ is the unit vector in the x-direction, and T is a scalar. In your handwritten answers, please indicate vectors (unit vectors) with an arrow (hat) above the symbol.

1. (5 points) The potential of a dipole, located at the origin, and pointing in the direction of the positive z-axis, is given by
$$V_{\text{dip}}(r,\theta) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{p}}{4\pi\epsilon_0 r^2}.$$
From this potential, calculate the electric field of the dipole in spherical coordinates.
Answer: Page 158 in the book. Take the negative gradient of V, in spherical coordinates, to get
$$\mathbf{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\hat{\boldsymbol{r}} + \sin\theta\boldsymbol{\theta})$$
+1 point to find the explicit θ dependence of the potential, then +4 to correctly take the gradient in spherical coordinates.

2. A point charge q is situated at a large distance r from a neutral atom of polarizability α.
   (a) (3 points) Find the dipole moment induced in the atom. What is the direction of this dipole?
   (Variation of problem 4.4) The field of q is $\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\boldsymbol{r}}$. This field leads to an induced dipole of the atom $\mathbf{p} = \alpha\mathbf{E} = \frac{\alpha q}{4\pi\epsilon_0 r^2}\hat{\boldsymbol{r}}$. It therefore points away from the charge.
   (b) (3 points) Is the force between the point charge and the atom attractive, repulsive or zero? Explain!
   There is an electric field at the location of the atom, which points in the direction of the dipole. It leads to an attractive force between the charge and the dipole.

3. Three charges are situated at the corners of a square (side a), as indicated in the figure.



   (a) (5 points) How much work does it take to bring in another charge $-2q$ from far away, and put it in the fourth corner?

   Answer: problem 2.31 a):
$$V = \frac{1}{4\pi\epsilon_0}\sum\frac{q_i}{r_{ij}} = \frac{1}{4\pi\epsilon_0}\left\{\frac{-q}{a} + \frac{q}{\sqrt{2}a} + \frac{-q}{a}\right\} = \frac{q}{4\pi\epsilon_0 a}(-2 + \frac{1}{\sqrt{2}}).$$

Therefore,

$$W_4 = -2qV = \frac{q^2}{4\pi\epsilon_0 a}(4-\sqrt{2})$$

Answer model: 3 points for calculating the potential correctly, 2 points for relating potential to work done correctly.

(b) (2 points) We observe the electric field generated by the four-charge arrangement that was assembled in the previous question. At a distance $r >> a$, the electric field is proportional to

A. r^{-1}
B. r^{-2}
C. r^{-3}
D. r^{-4}

Answer: There is a monopole contribution, and therefore the potential drops as r^{-2}.

4. Consider a parallel plate capacitor, with surface A, and spacing between the plates d. The top plate is charged using a power supply to $+Q$, the bottom is charged to $-Q$.

(a) (5 points) What is the capacitance of this parallel plate capacitor?
Example 2.11. The surface charge density $\sigma = Q/A$*, on both plates. Therefore the field between the plates is* $\frac{1}{\epsilon_0}\frac{Q}{A}$ *(see example 2.6). The potential difference is therefore* $V = \frac{Q}{A\epsilon_0}d$*, and therefore* $C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$.

(b) (5 points) After they are charged up, the plates are disconnected from the power supply, and then the distance between the plates is doubled. How do the capacitance, the voltage, the charge and the electric field change? Explain and quantify!
The charge is constant, because the plates are not connected to the power supply. The capacitance is halved, because the distance is doubled. Because the charge is constant, the voltage has to double. The electric field is constant - it is independent of the distance between the plates.

(c) (5 points) Next, a neutral slab of dielectric material with a dielectric constant $\epsilon_r = 20$ is placed in between the plates, completely filling the empty space (now with spacing $2d$). The plates are still not connected to the power supply. How do the capacitance, the voltage, the charge and the electric field change? Explain and quantify!
The electric field is reduced by a factor 20, due to the polarization of the dielectric material. The voltage is also a factor 20 lower. The charge remains the same, and therefore the capacitance is increased by a factor of 20.

5. A metal sphere of radius a carries a charge Q. It is surrounded, out to radius b, by linear dielectric material of permittivity ϵ.

(a) (3 points) Calculate the displacement inside and outside the metal sphere.
(example 4.5) Inside, $\mathbf{D} = \mathbf{E} = \mathbf{P} = 0$. *Outside,* $\mathbf{D} = \epsilon_0\mathbf{E}$*, and using a Gaussian surface we find* $\mathbf{D} = \frac{Q}{4\pi r^2}\hat{\boldsymbol{r}}$.

(b) (3 points) Calculate the electric field for $r > b$ and $a < r < b$.
(example 4.5) For $r > b$*, we have* $\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\boldsymbol{r}}$. *Inside the dielectric, it is reduced by* ϵ_r *giving* $\mathbf{E} = \frac{Q}{4\pi\epsilon r^2}\hat{\boldsymbol{r}}$.

(c) (3 points) Calculate the potential at the center of the sphere.
(example 4.5)

(d) (3 points) Calculate the polarization of the dielectric using the electric field.
(example 4.5)

(e) (3 points) Calculate the bound volume charge ρ_b in the dielectric and the surface charge σ_b on the inner and outer surface of the dielectric.
(example 4.5)

The End